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Arc. ° '	No. of Sec^{ds}.	sin = 4·685 +	Diff.	tan = 4·685 +	Diff.
2 35	9300	4278	19	8693	39
36	9360	4259	20	8732	38
37	9420	4239	19	8770	38
38	9480	4220	20	8808	39
39	9540	4200	19	8847	39
40	9600	4131	20	8886	40
41	9660	4161	20	8926	39
42	9720	4141	20	8965	39
43	9780	4121	20	9004	41
44	9840	4101	20	9045	40
45	9900	4081	20	9085	41
46	9960	4061	20	9126	41
47	10020	4041	21	9167	41
48	10080	4020	21	9208	41
49	10140	3999	21	9249	41
50	10200	3978	21	9290	42
51	10260	3957	20	9332	43
52	10320	3937	22	9375	41
53	10380	3915	21	9416	44
54	10440	3894	21	9460	42
55	10500	3873	22	9502	43
56	10560	3851	21	9546	43
57	10620	3830	23	9589	43
58	10680	3807	21	9632	44
2 59	10740	3786	22	9676	44
3 0	10800	3764		9720	

Watford, Herts, March 4, 1861.

From a Letter from Professor Hansen to the Astronomer Royal, dated Gotha, 1861, *Feb.* 2.

(*Translated from the German.*)

"You express in your honoured letter the wish that I would soon publish the calculation of my Lunar Perturbations, and I can assure you that I myself fully take part in that wish. I am even occupied, as far as my other labours permit, with the performance of this: a work of tolerable size is already finished, and I hope soon to be in a condition to draw up the manuscript for press for the first treatise. As far as I have yet advanced in the new calculations, I have found no differ-

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ence from the coefficients used in the Lunar Tables, which amounts to $0''\cdot1$; the new coefficients differ, at the most, from those of the Lunar Tables by a few hundredth parts of a second. My plan in this new investigation of the Lunar Perturbations stands firm, and cannot be altered.

"You wish further to know what I understand by the expression in my letter of 1854, Nov. 3, 'In my Lunar Tables I have provisionally used coefficients, which are not free from some empiricism,' and I will willingly communicate this to you. In the first place, I mean thereby the part of the motion of the line of nodes and of the line of apsides which is proportional to the time itself; and in the next place, the coefficient of the argument 8 V — 13 E; nothing further, unless also the determination of the coefficient of the parallactic equation by observations be called empiricism. In order to be able to calculate those coefficients by theory alone, with such accuracy that they could be employed in the tables without anything further, one would be obliged to calculate the coefficients of the periodical perturbations with far greater exactness than is otherwise demanded; and this I considered to be superfluous, as it would have engrossed very much time. For the rest, I have found the coefficient of 8 V — 13 E, by my last theoretical determination of it, by no means insensible, like Delaunay. Without the introduction of this coefficient, the observations show deviations at different epochs; but with the introduction of this, these deviations disappeared even to the last trace. I considered, therefore, its introduction as established, and reserve to myself a new theoretical determination of it, but cannot take this in hand until I shall have proceeded further in the new calculation of the remaining coefficients. I have, besides, some other inequalities of long period, which are caused by the planets; but as the coefficients of these inequalities are small, I have neglected them in the tables, in order to avoid too great extension.

"It is probably the recent publication of Delaunay which has led you, most highly-honoured colleague, to the inquiry which I have plainly answered above; and I see myself compelled, in relation thereto, to add that, in the comparison with the observations, I have taken in no old eclipse or other ancient observation. I have never gone beyond Bradley's observations, and nevertheless my tables represent the ancient observations with satisfactory agreement.

"I have already shown that the ancient eclipses cannot be satisfied by the secular variation of Adams and Delaunay (*Comptes Rendus*, tome l. No. 10). I have now made trial of it, omitting the inequality dependent upon 8 V — 13 E, but find the effect of this very trifling, and almost the same enormous differences. I would gladly have said nothing of this until my new calculation of the secular variation was finished; but with the inducement which you have given me, I cannot

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refrain from bringing forward what I have found about a year ago in relation to this. The method which Messrs. Adams and Delaunay have pursued in the calculation of the secular variation of the moon's mean longitude requires, in the co-ordinates or the elliptic elements of the moon, the introduction of terms of the following form:—

$$a\,t\,\begin{matrix}\sin\\ \cos\end{matrix}\left\{ig + i'g' + \mathrm{K}\right\} + \frac{b}{n}\,\begin{matrix}\cos\\ \sin\end{matrix}\left\{ig + i'g' + \mathrm{K}\right\} \ldots\ldots \quad (\mathrm{A})$$

where a and b are numerical coefficients, i and i' whole numbers, g and g' the mean anomalies of the Moon and of the Sun, K a function of the perihelion and of the nodes, and n the secular mean motion of the Moon. Against this nothing can be said; and Plana and Pontécoulant, who have attacked this principle, are in this respect in error. Methods, indeed, can be given in which the second class of the above-mentioned terms do not come into consideration; but if this matter be treated as Adams and Delaunay have done, these terms must be introduced. But—whence do these terms arise? They arise out of the following terms of the Sun's co-ordinates:—

$$a'\,t\,\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}i'g' + \frac{b'}{n}\,\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}i'g' \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathrm{B})$$

and the method which requires the complete taking into account of the terms (A) requires, of course, also the complete taking into account of the terms (B), and these are not small. The greatest of the coefficients denoted by b' exceeds 90″. Further, the part of the perturbation-function of the Moon depending upon the planets contains terms which may also contribute to the secular variation of the Moon's mean longitude: these are, however, smaller than those above.

"All the memoirs of Messrs. Adams and Delaunay on this matter which have come into my hands I have carefully looked through, but I have not been able to remark that regard had been taken of the above coefficients denoted by b', or of those depending upon the planets. Until I am convinced of the contrary, therefore, I must assume that these terms have not been taken into account; and I have in this, again, a ground for the suspicion that Delaunay may also have overlooked terms in the latest calculation of the coefficient of 8 V — 13 E.

"I can, besides, affirm that even in Adams' first memoir (received and read June 16, 1853) combinations have been overlooked* which, indeed, for the coefficient there calculated, accidentally destroy each other, but whose continued omission must necessarily make the terms of the higher orders incorrect. We find, in the memoir named, in the longitude,

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* In the original thus, "die zwar für den dort berechneten Coefficienten sich zufällig aufheben, aber deren fortgesetzte Uebergehung nothwendig die Glieder der höheren Ordnungen unrichtig machen muss."—Ed.

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$$+\frac{295}{24}m^2\frac{e'\,de'}{n\,dt}\cos(2\nu-2m\nu)-\frac{413}{48}m^2\frac{de'}{n\,dt}\cos(2\nu-2m\nu-c'm\nu)*$$

$$+\frac{59}{48}m^2\frac{de'}{n\,dt}\cos(2\nu-2m\nu+c'm\nu);*$$

while these coefficients are respectively =

$$+\frac{74}{3}m^2,\qquad -\frac{215}{48}m^2,\qquad +\frac{257}{48}m^2.$$

In the radius, we there find

$$+\frac{95}{12}m^2\frac{e'\,de'}{n\,dt}\sin(2\nu-2m\nu)-\frac{133}{24}m^2\frac{de'}{n\,dt}\sin(2\nu-2m\nu-c'm\nu)*$$

$$+\frac{19}{24}m^2\frac{de'}{n\,dt}(\sin 2\nu-2m\nu+c'm\nu);*$$

while these coefficients are respectively

$$+\frac{203}{12}m^2,\qquad -\frac{61}{24}m^2,\qquad +\frac{91}{24}m^2.$$

"It appears that the combination of the term $3\frac{de'}{n\,dt}\cos c'm\nu$ with the other entering terms has been overlooked.

"In conclusion, I must remark that I shall never be able to favour the development of the Lunar Perturbations in series according to the powers of m, unless it is proved,

"(1.) That these series converge;

"(2.) If this is the case, that the sum of the undeveloped terms of each one of these series can be assigned with sufficient approximation.

"That the first terms of a series decrease is no proof of the convergence of a series, for many diverging series can be brought forward in which this case occurs. It is also known that in slowly converging series the sum of the small terms not taken into account may often amount to something considerable."

Results of Observations of the Solar Eclipse of 1860, *July* 18, *made at the Royal Observatory, Greenwich, for determination of the Errors of the Tabular Elements of the Eclipse.* By G. B. Airy, Esq., Astronomer Royal.

Before leaving the Observatory for observation of the Totality of the Eclipse in Spain, I drew up a set of Instructions

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* In Prof. Hansen's manuscript the coefficients of these terms are written $\frac{e'\,de'}{n\,dt}$.—Ed.